10.28.22

LECTURE 29

- recall the velocity wrt t is $v(t) = r'(t)$
  - ↳ $v(t)$ is the instantaneous rate of change of arc length wrt
- therefore, ds/dt is called the speed of the particle
- $v(t) = \frac{ds}{dt} T(t)$
- speed is also the magnitude of $v(t)$, that is, $\|v(t)\|$

THEOREM If the position of a particle wrt t is $r(t)$, and s is an arc length parameter, then

$$a(t) = \frac{d^2s}{dt^2} T(t) + K(t)\left(\frac{ds}{dt}\right)^2 N(t)$$

- basically means that $a(t)$ lies in the plane of $T(t)$ and $N(t)$ and is always orthogonal to $B(t)$

EXAMPLE Prove the above theorem.

$v(t) = \frac{ds}{dt} T(t)$ we know $T'(t) = \|T'(t)\| N(t)$

$$v'(t) = \frac{d^2s}{dt^2} T(t) + \frac{ds}{dt} T'(t)$$

$$v'(t) = \frac{d^2s}{dt^2} T(t) + \frac{ds}{dt} \|T'(t)\| N(t)$$

Since we know $K(t) = \frac{\|T'(t)\|}{\|r'(t)\|}$, and $\|r'(t)\| = \frac{ds}{dt}$

$$v'(t) = \underbrace{\frac{d^2s}{dt^2} T(t)}_{\text{tangential direction}} + \underbrace{\left(\frac{ds}{dt}\right)^2 K(t) N(t)}_{\text{normal direction}}$$ 因.

- $a(t)$ is always orthogonal to $B(t)$ and lies in the plane of $N(t)$ and $T(t)$
- notation: $a_T = \frac{d^2s}{dt^2}$, $a_N = K(t) \cdot \left(\frac{ds}{dt}\right)^2$

$$\boxed{a(t) = a_T(t) T(t) + a_N(t) N(t)}$$

THEOREM If a particle has position of $r(t)$, then at each time t, the vectors v, a, and scalars K, a_T, a_N are related by:

$$a_T = \frac{v \cdot a}{\|v\|} \qquad a_N = \frac{\|v \times a\|}{\|v\|} \qquad K = \frac{\|v \times a\|}{\|v\|^3}$$

EXAMPLE Prove the above theorem.

1)

$a = a_T T(t) + a_N N(t)$

$$a \cdot T = a_T T \cdot T + a_N N \cdot T = a_T \|T\|^2 = a_T$$

we know $T = \frac{v}{\|v\|}$, so

$$a_T = a \cdot T$$

$$a_T = \frac{a \cdot v}{\|v\|}$$ 因

2) $a_N = \dfrac{\|v \times a\|}{\|v\|}$

$$a \times T = a_T T \times T + a_N N \times T = 0 + B(t) = B(t)$$

we know $B(t) = \dfrac{r'(t) \times r''(t)}{\|r'(t) \times r''(t)\|}$, so

$$a \times T = \frac{v \times a}{\|v \times a\|}$$

$a_N(t)N(t) = \left(\frac{ds}{dt}\right)^2 K(t)N(t)$

$a_N = \|v\|^2 \dfrac{\|v \times a\|}{\|v\|^3}$

$a_N = \left(\frac{ds}{dt}\right)^2 K(t)$

$a_N = \dfrac{\|v \times a\|}{\|v\|}$ ☒

$a_N = \|v\|^2 K(t)$

3) $K = \dfrac{\|r' \times r''\|}{\|r'\|^3} = \dfrac{\|v \times a\|}{\|v\|^3}$ ☒